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Less Conservative Consensus of Multi-agent Systems with Generalized Lipschitz Nonlinear Dynamics

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Abstract : *This paper addresses the problem of consensus of multi-agent systems with general Lipschitz nonlinear dynamics. The goal of this note was to introduce a design approach. We take advantage of the structure of the part of the non-linearity and inequality scaling technology. The advantage of the developed approach was that it was significantly less conservative than other previously published results for Lipschitz systems. A numerical example was presented to show the superiority of this letter.*
Keywords- *Consensus, Less Conservative Condition, Lipschitz Nonlinear Dynamics, Multi-agent Systems*

I. Introduction

Recently, consensus for a group of agents has became an important problem in the area of cooperative control of multi-agent systems and has been widely investigated by numerous researchers, due to its potential applications in such broad areas as satellite formation flying, sensor networks, and cooperative surveillance[1,2]. In [3] a distributed observer-type consensus protocol based on relative output measurements was proposed. In [4], a general framework of the consensus problems for networks of dynamic agents with fixed or switching topologies and communication time delays was established. Relative-state distributed consensus protocols was established in [5]. A distributed algorithm was proposed in [6] to achieve consensus in finite time. However the provided synthesis conditions which infeasible for systems with big Lipschitz constants restrict all these approaches, such as [6],[7],[8]. Then in [18] the authors introduced a generalized version of the Lipschitz condition which includes some structural knowledge of the system non-linearity.

This paper considers the problem of consensus of multi-agent systems with general Lipschitz nonlinear dynamics under a fixed topology. Consensus of multi-agent systems with general linear dynamics was researched in [3],[5],[14],[15],[19]. Especially, different static and dynamic consensus protocols are established in [3],[5],[14] which is difficult to tackle and implement by each agent in distributed fashion. Then, we design a distributed consensus protocol for the relative states and an adaptive law for adjusting the coupling weights between neighboring agents which were first proposed in [6]. One contribution of this paper is to extend the result of [3],[5],[6],[14],[20] to a generalized Lipschitz nonlinear system which includes the classical Lipschitz system as a special case. The goal of this note was to introduce a design approach. We take advantage of the structure of the part of the non-linearity and inequality scaling technology. The advantage of the developed approach was that it was significantly less conservative than other previously published results for Lipschitz systems.

The rest of this paper was organized as follows. Some useful preliminaries results were reviewed in Section 2. The consensus problems of multi-agent systems with generalized Lipschitz non-linear dynamics using distributed adaptive protocols were investigated in Section 3. Extensions to Lipschitz non-linearity dynamics were studied in Section 4. In Section 5, we use a simulative example to illustrate the applications of our consensus algorithm. Section 6 concludes the paper.

II. Preliminaries

A. Graph theory notions

Let $R^{n\times n}$ be the set of $n\times n$ real matrices, $C^{n\times n}$ be the set of $n\times n$ complex matrices. Denote by $\mathbf{1}$ ($\mathbf{1}\in R^{P}$) a column vector with all entries equal to one. The superscript T means transpose for real matrices. I_P represents the identity matrix of dimension P. For any matrices, if not explicitly stated, were assumed to have compatible dimensions. Diag $(A_1,...,A_n)$ represents a block-diagonal matrix with matrix $A_i, i=1,...,n$, on its diagonal .The matrix inequality $A>(\geq)B$ means that A and B were square Hermitian matrices and that $A-B$ was positive (semi-) definite. $A\otimes B$ denotes the Kronecker product of matrices A and B. For a vector x, let $\|x\|$ denote its 2-norm. An undirected graph $G=(\nu,\varepsilon)$,where $\nu=\{v_1,\dots,v_N\}$ was the set of nodes(i .e ,agents),and $\varepsilon\subset\nu\times\nu$ was the set of edges(i .e, communication links).An edge $(v_i,v_j)(i\neq j)$ means that agents v_i and v_j can obtain information from each other. A path between distinct nodes v_i and v_l was a sequence of edges of the form $(v_k,v_{k+1}),k=i,\dots,l-1$. An undirected graph was connected if there exists a path between every pair of distinct nodes, otherwise was disconnected. A directed graph G was a pair (ν,ε), where $\nu=\{v_1,\dots,v_N\}$ was a non-empty finite set of notes and $\varepsilon\subset\nu\times\nu$ was the set of edges. For an edge $(v_i.v_j)$, node v_i was called the parent node ,node v_j was the child node, $v_i.v_j$ were adjacent. A directed graph contains a directed spanning tree if there exists a node called the root, which has no parent node, such that the node has a directed path to every other node in the graph. A directed graph was strongly connected if there was a directed path from every node to every other node. A directed graph has a directed spanning tree if it was strongly connected , but not vice versa.

The adjacency matrix $A=[a_{ij}]\in R^{N\times N}$ associated with the directed graph G was defined by $a_{ii}=0,a_{ij}=1$ if $(v_j,v_i)\in\varepsilon$ and $a_{ij}=0$ otherwise. Adjacency matrix A associated with the undirected graph was defined by $a_{ii}=0,a_{ji}=a_{ij}=1$ if $(v_j,v_i)\in\varepsilon$ and $a_{ji}=a_{ij}=0$ otherwise. Laplacian matrix $L=[L_{ij}]\in R^{N\times N}$ was defined as $L_{ii}=\sum_{j\neq i}a_{ij}$ and $L_{ij}=-a_{ij},i\neq j$.

B. Lemmas:

Lemma 1[8,9]: Zero was an eigenvalue of L with 1 as a right eigenvector and all non-zero eigenvalues have positive real parts .Furthermore ,zero was a simple eigenvalue of L if and only if the graph G has a directed spanning tree .

Lemma2[17]: For any vectors $a,b\in R^n$ and scalar $\varepsilon>0$,we have $2a^Tb\le\varepsilon a^Ta+\varepsilon^{-1}b^Tb$.

III. Consensus of multi-agent systems with generalized Lipschitz non-linear dynamics .

Consider a group of N identical agents with generalized Lipschitz nonlinearity dynamics. The dynamics of the i -th agent were described by

$$\dot{x}_i=A_ix_i+Df(Hx_i)+Bu_i\quad i=1,\,..N, \tag{1}$$

where $x_i\in R^n$ was the state, $u_i\in R^p$ was the control input, and A,B,D,H were constant matrices with compatible dimensions, and nonlinear function $Df(Hx_i)$ was assumed to satisfy the following general form Lipschitz condition which was defined in [18] as follows

$$\tilde{f}^TW\tilde{f}\le\tilde{x}^TR\tilde{x} \tag{2}$$

where the W and R were positive definite symmetric matrices . $\tilde{f}=f(Hx_i)-f(Hx_j),\tilde{x}=x_i-x_j$. It can be seen that any non-linear function $Df(Hx)$ was a Lipschitz function with $\gamma=\sqrt{\sigma_{\max}(R)/\sigma_{\min}(W)}$ which called Lipschitz constant. Any Lipschitz non-function $Df(Hx)$ can satisfy condition (2) with $W=I,R=\gamma^2I$. (2) can also be rewritten as a generalized form$\|K_W\tilde{f}\|\le\|K_R\tilde{x}\|,K_W$ 、 K_R were two positive definite matrices. $K_W=\sqrt{W},K_R=\sqrt{R}$.

The communication topology among the agents was represented by an undirected graph $G=(\nu,\varepsilon)$, where $\nu=\{1,\dots,N\}$ was the set of nodes(i .e ,agents), and $\varepsilon\subset\nu\times\nu$ was the set of edges(i .e, communication links). An edge $(v_i,v_j)(i\neq j)$ means that agents v_i and v_j can obtain information from each other. A path between distinct nodes v_1 and v_l was a sequence of edges of the form $(v_k,v_{k+1}),k=1,\dots,l-1$. An undirected graph was connected if there exists a path between every pair of distinct nodes , otherwise was disconnected.

A variety of static and dynamic consensus protocols have been proposed to reach consensus for agents with dynamics given by (1) e.g, in [1]–[5]. For instance, a dynamic consensus protocol based on the relative states between neighboring agents was given in [6] as follows

$$\dot{c}_{ij}=k_{ij}a_{ij}(x_i-x_j)^T\Gamma(x_i-x_j)\quad u_i=F\sum_{j=1}^{N}c_{ij}a_{ij}(x_i-x_j) \tag{3}$$

where a_{ij} was (i, j)-th entry of the adjacency matrix A associated with G , $k_{ij} = k_{ji}$ were positive constants, c_{ij} denotes the time-varying coupling weight between agents v_i and v_j with $c_{ij}(0) = c_{ij}(0)$,and $F \in R^{p \times n}$ and $\Gamma \in R^{n \times n}$ were the feedback gain matrices.

Theorem 1 :Solve the LMI :

$$\begin{pmatrix} AP + PA^T + \varepsilon DW^{-1}D^T - \varphi BB^T & PH^T \\ HP & -\varepsilon R^{-1} \end{pmatrix} < 0 \tag{4}$$

to get a matrix $P > 0$ and a scalar $\varphi > 0$ .Then the N agent described by (1) reach global consensus under the protocol (3) with $F = -B^T P^{-1}$ and $\Gamma = P^{-1} BB^T P^{-1}$ for any connected communication graph G .

Proof: As argued in the Section 2 ,it follows that $c_{ij}(t) = c_{ji}(t), \forall t \geq 0.$ Using (3) for (1),we obtain the closed-loop network dynamics as

$$\dot{x}_i = Ax_i + Df(Hx_i) + BF\sum_{j=1}^{N} c_{ij} a_{ij}(x_i - x_j)$$

$$\dot{c}_{ij} = k_{ij} a_{ij}(x_i - x_j)^T \Gamma (x_i - x_j) \quad i = 1, ..., N \tag{5}$$

Letting and $e = [e_1^T, ..., e_N^T]^T$, we get $e = [(I_N - (1/N)11^T) \otimes I_n]x$ .We can see that e satisfies the following dynamics:

$$\dot{e}_i = Ae_i + Df(Hx_i) - \frac{1}{N}\sum_{j=1}^{N} Df(Hx_i) + \sum_{j=1}^{N} (\tilde{c}_{ij} + \beta) a_{ij} BF(e_i - e_j)$$

$$\dot{\tilde{c}}_{ij} = k_{ij} a_{ij}(e_i - e_j)^T \Gamma (e_i - e_j) \quad i = 1, ..., N \tag{6}$$

where $c_{ij} = \tilde{c}_{ij} + \beta$ and β was a positive constant. Consider the Lyapunov function candidate

$$\dot{V}_1 = 2\sum_{i=1}^{N} e_i^T P^{-1} \dot{e}_i + \sum_{i=1}^{N} \sum_{j=1, j \neq i}^{N} \frac{\dot{\tilde{c}}_{ij} \tilde{c}_{ij}}{k_{ij}} \tag{7}$$

the time derivative of V along the trajectory of (8) as follows:

$$\dot{V}_1 = 2\sum_{i=1}^{N} e_i^T P^{-1} \dot{e}_i + \sum_{i=1}^{N} \sum_{j=1, j \neq i}^{N} \frac{\dot{\tilde{c}}_{ij} \tilde{c}_{ij}}{k_{ij}}$$

$$= 2\sum_{i=1}^{N} e_i^T P^{-1} A e_i - 2\beta \sum_{i=1}^{N} \sum_{j=1}^{N} L_{ij} e_i^T P^{-1} BB^T P^{-1} e_j + 2\sum_{i=1}^{N} e_i^T P^{-1} D[f(Hx_i) - f(H\overline{x}_i) + f(H\overline{x}_i) - \frac{1}{N}\sum_{j=1}^{N} f(Hx_j)] \tag{8}$$

According to the equivalent form of Lipschitz condition (2) and Lemma 2 we obtain

$$\begin{aligned} 2e_i^T P^{-1} D[f(Hx_i) - f(H\overline{x}_i)] &= 2e_i^T P^{-1} DW^{-\frac{1}{2}} W^{\frac{1}{2}} [f(Hx_i) - f(H\overline{x}_i)] \\ &\leq \varepsilon e_i^T P^{-1} DW^{-1} D^T P^{-1} e_i + \frac{1}{\varepsilon} [f(Hx_i) - f(H\overline{x}_i)]^T W [f(Hx_i) - f(H\overline{x}_i)] \\ &\leq \varepsilon e_i^T P^{-1} DW^{-1} D^T P^{-1} e_i + \frac{1}{\varepsilon} e_i^T H^T R H e_i \\ &= e_i^T [\varepsilon P^{-1} DW^{-1} D^T P^{-1} + \frac{1}{\varepsilon} H^T R H] e_i, \forall \varepsilon > 0 \end{aligned} \tag{9}$$

As $\sum_{i=1}^{N} e_i^T = 0$, then

$$\sum_{i=1}^{N} e_i^T P^{-1}[f(H\bar{x}_i) - \frac{1}{N}\sum_{j=1}^{N} f(Hx_j)] = 0 \tag{10}$$

Set up $\hat{e}_i = P^{-1}e_i$, and $\hat{e} = [e_i^T,...,e_N^T]^T$, based on (9)and(10)we can draw from (8)that

$$\begin{aligned} &= \sum_{i=2}^{N} \zeta_i^T [AP + PA^T + \varepsilon DW^{-1}D^T + \frac{1}{\varepsilon} PH^T RHP - 2\beta\lambda_i BB^T]\zeta_i \triangleq S(\zeta) \\ &= \hat{e}_i^T [I_N \otimes (AP + PA^T + \varepsilon DW^{-1}D^T + \frac{1}{\varepsilon} PH^T RHP) - 2\beta L \otimes BB^T]\hat{e}_i \end{aligned} \tag{11}$$

Set up $U \in R^{N\times N}$ be the unitary matrix satisfying $U^T LU = \Lambda = diag(0, \lambda_2, ..., \lambda_N)$,set up $\zeta \triangleq [\zeta_1^T,...,\zeta_N^T]^T = (U^T \otimes I_N)\hat{e}$, then $\zeta_1 = [(1T/\sqrt{N}) \otimes P^{-1}]e = 0$,from (11)we have

$$\begin{aligned} V_1 &\le \zeta^T [I_N \otimes (AP + PA^T + \varepsilon DW^{-1}D^T + \frac{1}{\varepsilon} PH^T RHP) - 2\beta\Lambda \otimes BB^T]\zeta \\ &= \sum_{i=2}^{N} \zeta_i^T [AP + PA^T + \varepsilon DW^{-1}D^T + \frac{1}{\varepsilon} PH^T RHP - 2\beta\lambda_i BB^T]\zeta_i \triangleq S(\zeta) \end{aligned} \tag{12}$$

We choose sufficiently big β so that $2\beta\lambda_i \ge \varphi, i = 2,..., N$, then

$$\begin{aligned} &AP + PA^T + \varepsilon DW^{-1}D^T + \frac{1}{\varepsilon} PH^T RHP - 2\beta BB^T \\ &\le AP + PA^T + \varepsilon DW^{-1}D^T + \frac{1}{\varepsilon} PH^T RHP - \varphi BB^T < 0 \end{aligned} \tag{13}$$

We get inequality (4) from (14) by using the Schur complement Lemma[12]. we know that $S(\zeta) \le 0.$ $\dot{V}_1 \le 0$, so $V_1(t)$ and every $\tilde{c}_{ij}$ were bounded. And from (5) we know that $\tilde{c}_{ij}$ was monotonically increasing, so $\tilde{c}_{ij}$ converge to finite value and c_{ij} as well. By using LaSalle-Yoshizawa Theorem [13] we get $\lim_{t\to\infty} S(\zeta) = 0$,so $\lim_{t\to\infty} \zeta_i \to 0, i = 2,..., N$, $\zeta_1 \equiv 0$. In conclusion, $\lim_{t\to\infty}, e(t) \to 0$. so, we achieve the proof .

Remark 1: Equation (2) was a generic form Lipschitz condition. Any Lipschitz non-function $f(Hx)$ can satwasfy equation (2) with $W = I, R = \gamma^2 I$ which was first define in [18] includes some structural knowledge of the system non-linearity for observer design. Then, it was used to consensus of multi-agent systems with non-linear dynamics

Deduction to Lipschitz non-linearity dynamics

This subsection considers the consensus problem of the agents in (1) under the adaptive protocol (3). The communication topology among the agents was represented by an undirected graph. The nonlinear function $Df(Hx_i)$ was assumed to satisfy the Lipschitz condition with a Lipschitz constant $\gamma > 0$, i.e.

$$\| f(Hx) - f(Hy) \| \le \gamma \| H(x - y) \| \qquad \forall x, y \in R^n \tag{14}$$

Theorem 2 :Solve the LMI :

$$\begin{pmatrix} AP + PA^T + \delta DD^T - \kappa BB^T & \gamma PH^T \\ \gamma HP & -\delta I \end{pmatrix} < 0 \tag{15}$$

To get a matrix $P > 0$ and a scalar $\kappa > 0$ .Then the N agent described by (1) reach global consensus under the protocol (3) with $F = -B^T P^{-1}$ and $\Gamma = P^{-1}BB^T P^{-1}$ given as in Theorem 1 for any connected communication graph G .The proof of Theorem 1can be shown by following similar steps in proving Theorem 2

Remark 2:Theorem 2 tends Theorem 1 to the case with Lipschitz non-linear dynamics. When $W = I, R = \gamma^2 I$, Theorem 1will reduce to Theorem 2.Theorem 2 shows that the agents in (1) under the adaptive protocol (3) can also get consensus with the nonlinear function satisfying the Lipschitz condition.

IV. Numerical comparisons and simulation examples

In order to evaluate our design methodology, we present in this Section a numerical example to validate the effectiveness of the theoretical results. In fact, we show through this example that our approach was the best method that takes into account the structure of the nonlinearity in detail.

Consider a network of single-link manipulators with revolute joints actuated by a DC motor. The dynamics of the i-th manipulator was described by (3) with

$$x_i = \begin{bmatrix} x_{i1} \\ x_{i2} \\ x_{i3} \\ x_{i4} \end{bmatrix} \qquad A = \begin{bmatrix} 0 & 1 & 0 & 0 \\ -48.6 & -1.25 & 48.6 & 0 \\ 0 & 0 & 0 & 10 \\ 1.95 & 0 & -1.95 & 0 \end{bmatrix}$$

$$B = [0 \quad 21.6 \quad 0 \quad 0]^T$$
$$D = [0 \quad 0 \quad 0 \quad -0.333]^T$$
$$H = [0 \quad 0 \quad 1 \quad 0]$$
$$Df(Hx_i) = [0 \quad 0 \quad 0 \quad -0.333\sin(x_{i3})]^T$$

Clearly, $f(Hx_i)$ satisfies (4) with a Lipschitz constant $\gamma = 0.333$, solving the LMI (9) by using the LMI toolbox of Matlab gives the feasible solution as

$$\Gamma = \begin{bmatrix} 3239 & 500 & -2579 & 6459 \\ 500 & 77 & -398 & 998 \\ -2579 & -398 & 2054 & -5144 \\ 6459 & 998 & -5144 & 12881 \end{bmatrix}$$

$$\delta = 0.8088, \kappa = 0.3634$$

$$F = [-56.68 \quad -8.767 \quad 45.15 \quad -113.1]$$

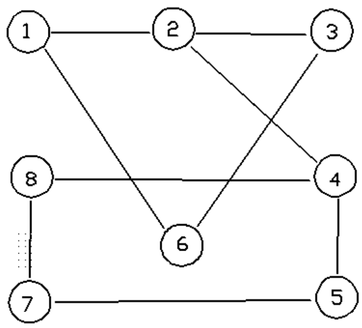

Fig 1 .Communication topology.

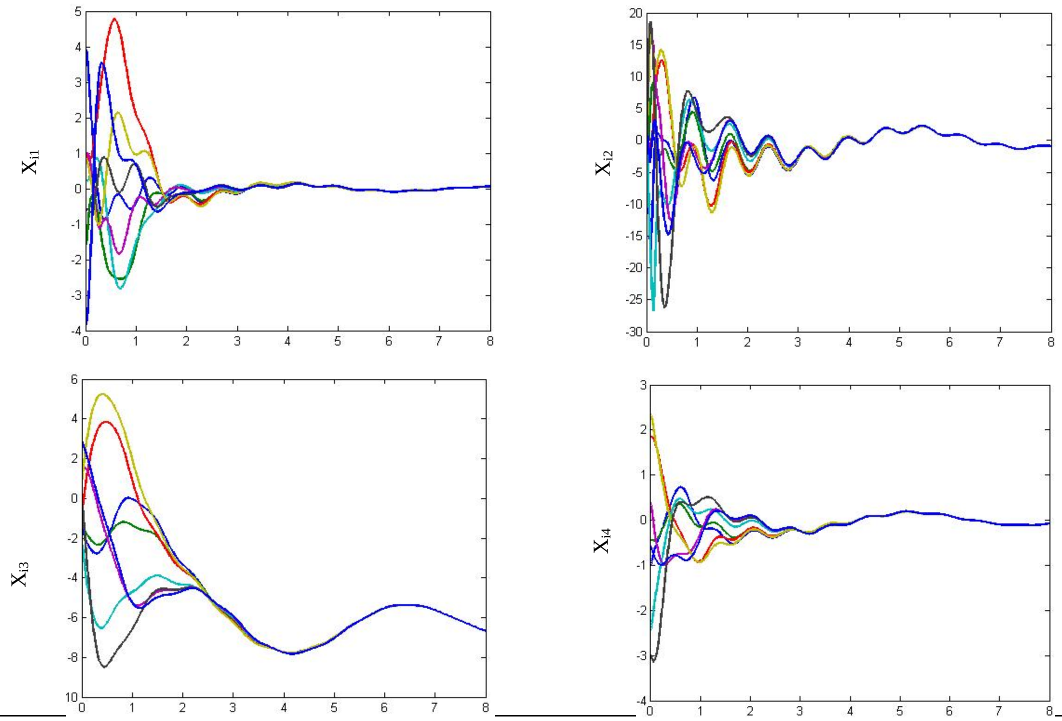

Fig 2. States of the eight manipulators under (3)

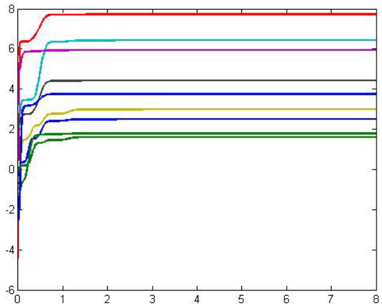

Fig 3 . Coupling weights c_{ij}.

In order to illustrate Theorem 2 ,we let the communication graph G as in fig 1. G was connected and undirected. There were eight manipulators. In other words, $i, j = 1,...,8$ in (1) , $k_{ij} = k_{ji} = 1$, $c_{ij}(0) = c_{ji}(0)$. The coupling weighs were as Fg 2 . They converge to steady values. The states of the manipulators which satisfies (3) were as Fg 3. They also get consensus.

Table1 .Comparwason of maximum Lipschitz constant for various LMI design techniques

Method	LMI(14)of L. Zhongkui(2013)	LMI(21)
$\gamma_{\max}$	0.34	$\approx 10^6$

We focus our study on the conservation of the states LMI approach. The comparison between different LMI conditions in Table 1 shows that our approach is significantly less conservative than other previously published results for Lipschitz systems.

V. Conclusions

We have studied the adaptive consensus problem of multi-agent systems with generalized Lipschitz nonlinear dynamics. A less conservative adaptive consensus condition has been proposed by carefully considering the structure information of nonlinearities and using a sharp inequality to deal with the generalized Lipschitz condition. It can significantly reduce the conservatism in some existing adaptive consensus results for Lipschitz nonlinear system， which is verified through a numerical example.

VI. Acknowledgement

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